RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIFTH SEMESTER EXAMINATION, DECEMBER 2012

THIRD YEAR

MATHEMATICS (Honours)

Date : 17/12/2012

Time : 11 am – 3 pm

Paper : V

Full Marks : 100

Use separate answer-book for each group

Group-A

Answer **any five** questions: 5x10=50

1. a) Let H be a subgroup of a group G such that [G : H] = 2. Prove that H is normal in G. 2
   b) Let H be a subgroup of a group G. If $x^2 \in H \forall x \in G$, prove that H is a normal subgroup of G and G/H is commutative. 3
   c) Let G be a group. If $G/Z(G)$ is cyclic, prove that G is abelian. Use it to show that the centre of S_3 is trivial. 5

2. a) State and prove sylow's First Theorem. 6
   b) Prove that $(R^+, \bullet)$ is the only subgroup of $(R^*, \bullet)$ of finite index, where $R^* = R - \{0\}$ and $R^+ = \{x \in R : x > 0\}$. 4

3. a) Prove that the group $4Z/12Z$ is isomorphic to Z_3. 3
   b) Show that a group G of order 231 contains a Sylow 11-subgroup which is is normal in G. 2
   c) Show that a group of order 42 cannot be simple. 3
   d) Is every abelian group of order 6 cyclic? Justify your answer. 2

4. a) Let H and K be two finite cyclic groups of order m and n respectively. Prove that the direct product H x K is a cyclic group if and only if gcd (m, n)=1. 5
   b) If G is a non-commutative group of order p^3, where p is a prime integer, then prove that |Z(G)|=p. 5

5. a) Show that if G is a finite p-group with |G|>1 then Z(G) is non-trivial. 3
   b) Show that there does not exist an onto homomorphism from the Klein's 4-group to the group $(Z_4,+)$. 2
   c) Prove that every group of order 14 contains only one normal subgroup of order 7. 5

6. a) Let G be a commutative group of order n. If gcd (m,n)=1, prove that the mapping $\phi : G \to G$ defined by $\phi(x) = x^m, x \in G$ is an automorphism. 5
   b) Show that no group of order 56 is simple. 5

7. a) Prove that any integral domain can be embedded in a field. 6
   b) Show that the identity automorphism is the only automorphism of the field R of all real numbers. 4

8. a) Prove that the rings $Z[\sqrt{3}]$ and $Z[\sqrt{5}]$ are not isomorphic. 3
   b) Show that the function $f : Z_6 \to Z_{10}$ defined by $f([a])=5[a]$ for all $a \in Z_6$ is a ring homomorphism of Z_6 into Z_{10}. 3
   c) Let R be a commutative ring with identity and A, B be two distinct maximal ideals of R. Show that $AB = A \cap B$. 4

Group-B

Answer any six questions from question no. 9 to question no. 17

9. Show that the error in approximating $f(x)$ by the interpolation polynomial using distinct interpolating points $x_0, x_1, \cdots, x_n$ is of the form $(x-x_0)(x-x_1)\cdots(x-x_n)\frac{f^{n+1}(\xi)}{\lfloor n+1}$, where $\min\{x, x_0, x_1, \cdots x_n\} < \xi < \max\{x, x_0, x_1, \cdots, x_n\}$. 5

10. a) Establish the uniqueness of interpolating polynomials. 2

b) Find the polynomial of degree 2 or less, such that $f(0)=1,\quad f(1)=3,\quad f(3)=55$ using the Newton divided difference interpolation. 3

11. a) The function $y(x)=\sqrt[3]{x}$ is tabulated below.

x:	5600	5700	5800	5900	6000
y:	17.75808	17.86316	17.96702	18.06969	18.17121

Compute $\sqrt[3]{5860}$ by starlings formula. 3

b) What is the speciality of Hermit interpolating polynomial? 2

12. a) From the following table find the value of $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ at the pt. x = 1.5.

x:	1.5	2.0	2.5	3.0	3.5	4.0
y:	3.375	7.000	13.625	24.000	38.875	59.000

3

b) Show that the Simpson's one third rule gives exact results if it is applied to any polynomial of degree 3. 2

13. Describe Newton-Raphson method for computing a simple root of $f(x)=0$. Give its geometrical interpretation. 4+1

14. Describe Gaussian elimination method for numerical solution of system of linear equation. What is the necessity of the pivoting Process involved in it? 4+1

15. Find the value of $y(0.4)$ using Runge-Kulta method of fourth order with h=0.2 given that $\frac{dy}{dx}=\sqrt{x^2+y},\quad y(0)=0.8$ correct up to four decimal places. 5

16. Describe the power method to calculate numerically greatest eigen value of a real square matrix of order n. 5

17. Use Picard's method to compute $y(0.1)$ from the differential equation $\frac{dy}{dx}=x+y;\quad y=1,\quad when\, x=0$. 5

Answer <u>any four</u> questions from question no. 18 to question no. 23

18. a) let x and n be positive integers such that $1+x+x^2+\cdots+x^{n-1}$ is a prime number. Then show that n is a prime number. 2

b) If g is the greatest common divisor of integers b and c, then prove that $g=bx_0+cy_0$ where $x_0, y_0 \in \mathbb{Z}$. 3

19. a) Show that if p and q are distinct primes, then $p^{q-1}+q^{p-1}\equiv 1 \pmod{pq}$. 3

b) Prove that the sum of the squares of two odd integers can not be a perfect square. 2

20. a) Find all solutions of the system

$$\left.\begin{array}{l} x\equiv 2 \pmod 4 \\ x\equiv 3 \pmod 5 \\ x\equiv 1 \pmod 7 \end{array}\right\}$$

2

b) State and prove Euclid's theorem. 3

21. a) Show that n is a perfect square if and only if the number of positive divisors of n is odd. 3

b) Define Mobius μ -function and state the Mobius inversion law. 2

22. a) Find the remainder when $\underline{|15}$ is divided by 17 1

b) Prove that there are infinitely many primes of the form 4n – 1 where n is some positive integer. 4

23. a) Find the positive integral solutions of the equation $4x+6y=8$. 3

b) If p is prime and k is a positive integer then prove that $\phi(p^k)=p^k\left(1-\frac{1}{p}\right)$ where ϕ is the Euler's ϕ function. 2